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PHYSICAL RELATIONS FOR PROBLEMS OF IMPACT LOADING AND UNSTEADY DEFORMATION OF COMPOSITE STRUCTURES

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A problem of determining elastic and viscous characteristics of composite materials, necessary and sufficient for choosing physical relations in solving problems of impact loading with low impact velocities (*up to* 200 m/sec) *and unsteady deformation in the range of strain rates within* 10^2 *sec*$^{-1}$ *for multilayer beams, plates, and shells, is considered.*

Key words: *composite materials, composite structures, beams, plates, shells, impact loading, unsteady deformation.*

Introduction. Aviation, rocketry, ship building, and other branches of industry widely apply multilayer composite materials (CMs), such as fiberglass plastic, carbon plastic, organic plastic, etc., with different directions of reinforcing fibers and volume fractions of the polymer binder.

The characteristics of unidirectional layers are used as a basis for calculating the effective elastic characteristics of a multilayer packet of the composite. Thus, multilayer shells, plates, and beams are characterized by effective rigidities, which, in turn, are determined by the elastic characteristics of rigidities of monolayers. All currently available methods of determining the effective characteristics of multilayer composites are based on the fundamental works by Lekhnitskii [1] and Ambartsumyan [2]; these methods were developed by Russian and foreign scientists (see [3–6], etc.), in particular, for dynamic problems (see, e.g., [7, 8], etc.).

Owing to the presence of a polymer binder, composite materials possess viscoelastic properties to a smaller or greater extent, which are manifested already under quasi-static loading; under impact loading, these properties becomes rather important. For this reason, a new research field was explored: identification of effective strain characteristics of models of viscoelastic deformation of composite structures under unsteady loading (see [8, 9], etc.).

At the same time, there exist the so-called unsteady methods of experimental identification of dynamic elastic constants of composite materials, based on measuring the velocities of longitudinal and flexural deformation waves [10, 11]. Experimental data on the degree of decay of these waves can be used for determining the dynamic viscous constants. In this case, the only restriction is the presence of layers of multilayer composites in amounts sufficient for realization of longitudinal impact loading of the samples. Preliminary experiments showed that there should be more than 12 layers in materials such as carbon plastic and fiberglass plastic, which is typical, for instance, for structures used in aviation.

Determination of a necessary and sufficient set of elastic and viscous constants of composite materials serves as a basis for choosing physical relations in solving problems of impact loading and unsteady deformation of elements of composite structures, such as multilayer beams, plates, and shells.

1. Physical Relations for Multilayer Shells, Plates, and Beams. Carbon plastic and fiberglass plastic materials are layered fiber-reinforced composite materials used to form multilayer structures, such as shells, plates, and beams. The layers of the CM packet whose structure is illustrated in Fig. 1 have different values of rigidity; therefore, the elastic properties of the entire packet are characterized by a rigidity tensor A. Hooke's law

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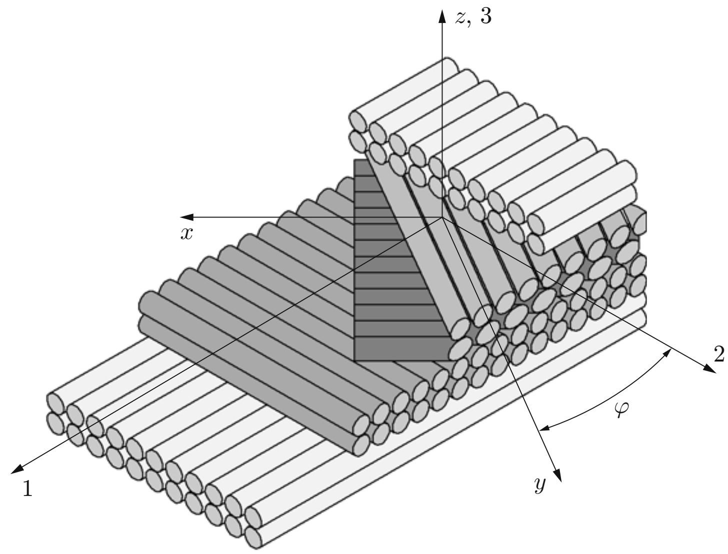

Fig. 1. Calculation scheme of a multilayer packet of a composite material for shells, plates, and beams.

for an arbitrary layer has the form

$$\sigma^{ij} = A^{ijkl} e_{kl} \qquad (i, j, k, l = 1, 2, 3), \tag{1}$$

where σ^{ij} are the components of the stress tensor, e_{kl} are the components of the strain tensor, and A^{ijkl} are the components of the rigidity tensor of the layer.

Below we derive relations for effective rigidities of multilayer shells with become simplified for multilayer plates and beams. The coordinate system (x_1, x_2, x_3) is used here. For cylindrical shells, x_1 is the coordinate in the meridional direction, x_2 is the coordinate in the circular direction, and x_3 is the coordinate along the normal to the shell mid-surface. For rectangular plates, (x_1, x_2, x_3) is a Cartesian coordinate system. In the problem of beam flexure, x_1 is the longitudinal coordinate and x_3 is the coordinate along the normal to the beam mid-surface.

For the cylindrical shells considered, we use the linear kinematic relations for gently sloping shells on the basis of the shear theory, such as Timoshenko's theory, in the following form [12]:

$$e_{\alpha\beta} = \Omega_{\alpha\beta} + x_3 \chi_{\alpha\beta}, \qquad e_{\alpha 3} = \Omega_{\alpha 3}, \qquad e_{33} = \Omega_{33} \qquad (\alpha, \beta = 1, 2). \tag{2}$$

Here $e_{\alpha\beta}$ are the components of the strain tensor of the shell, $\Omega_{\alpha\beta}$ are the components of the symmetric strain tensor of the mid-surface, $\Omega_{\alpha 3}$ are the shear strains, Ω_{33} are the normal strains, and $\chi_{\alpha\beta}$ are the components of the symmetric tensor of curvature of the shell mid-surface.

The expressions for the forces, moments, and shear forces have the form

$$\begin{gathered} N^{\alpha\beta} = \int\limits_{-H/2}^{H/2} \sigma^{\alpha\beta}\, dx_3, \qquad N^{33} = \int\limits_{-H/2}^{H/2} \sigma^{33}\, dx_3, \\ M^{\alpha\beta} = \int\limits_{-H/2}^{H/2} \sigma^{\alpha\beta} x_3\, dx_3, \qquad Q^{\alpha 3} = \int\limits_{-H/2}^{H/2} \sigma^{\alpha 3}\, dx_3 \qquad (\alpha, \beta = 1, 2). \end{gathered} \tag{3}$$

Here $N^{\alpha\beta}$ are the components of the tensor of the membrane forces of the shell, N^{33} are the components of the tensor of the normal squeezing force, $M^{\alpha\beta}$ are the components of the tensor of moments, and $Q^{\alpha 3}$ are the components of the tensor of the shear forces of the shell.

Substituting Eqs. (1) and (2) into relations (3), we obtain the physical relations

$$N^{\alpha\beta} = \Theta^{\alpha\beta\gamma\delta} \Omega_{\gamma\delta} + \Sigma^{\alpha\beta\gamma\delta} \chi_{\gamma\delta} + \Theta^{\alpha\beta 33} \Omega_{33}, \qquad N^{33} = \Theta^{3333} \Omega_{33} + \Theta^{33\alpha\beta} \Omega_{\alpha\beta} + \Sigma^{33\alpha\beta} \chi_{\alpha\beta},$$

$$M^{\alpha\beta} = \Sigma^{\alpha\beta\gamma\delta}\Omega_{\gamma\delta} + \Xi^{\alpha\beta\gamma\delta}\chi_{\gamma\delta} + \Sigma^{\alpha\beta 33}\Omega_{33}, \qquad Q^{\alpha 3} = 2\Theta^{\alpha 3\beta 3}\Omega_{\alpha 3} \qquad (\alpha,\beta,\gamma,\delta = 1,2),$$

where Θ, Σ, and Ξ are the tensors of the membrane, membrane–flexural, and flexural rigidities of the shell:

$$(\Theta, \Sigma, \Xi) = \int\limits_{-H/2}^{H/2} A(1, z, z^2)\, dz. \tag{4}$$

Integrating Eq. (4), we find the expressions for the effective rigidities of a multilayer shell, which include the rigidities of individual layers A_k:

$$\begin{aligned}
\Theta^{\alpha\beta\gamma\delta} &= \sum_{k=1}^{K} A_k^{\alpha\beta\gamma\delta}(z_{k+1} - z_k), \qquad & \Theta^{\alpha\beta 33} &= \sum_{k=1}^{K} A_k^{\alpha\beta 33}(z_{k+1} - z_k),\\
\Theta^{3333} &= \sum_{k=1}^{K} A_k^{3333}(z_{k+1} - z_k), \qquad & \Theta^{\alpha 3\beta 3} &= \sum_{k=1}^{K} A_k^{\alpha 3\beta 3}(z_{k+1} - z_k),\\
\Sigma^{\alpha\beta\gamma\delta} &= \frac{1}{2}\sum_{k=1}^{K} A_k^{\alpha\beta\gamma\delta}(z_{k+1}^2 - z_k^2), \qquad & \Sigma^{\alpha\beta 33} &= \frac{1}{2}\sum_{k=1}^{K} A_k^{\alpha\beta 33}(z_{k+1}^2 - z_k^2),\\
& & \Xi^{\alpha\beta\gamma\delta} &= \frac{1}{3}\sum_{k=1}^{K} A_k^{\alpha\beta\gamma\delta}(z_{k+1}^3 - z_k^3).
\end{aligned} \tag{5}$$

Using the formulas for the tensors of the effective rigidities of a multilayer shell (5), we can find the effective rigidities for multilayer plates and beams.

In the matrix recording of the physical relations, we use the following notations for the components of the effective rigidity tensors: $11 \to 1$, $22 \to 2$, $33 \to 3$, $23 \to 4$, $13 \to 5$, and $12 \to 6$. In the matrix recording, the physical relations (1) have the form

$$\{\sigma\} = [D]\{\varepsilon\}.$$

Here $\{\varepsilon\}^{\mathrm{t}}$ is the strain vector:

— for shells and plates,

$$\{\varepsilon\}^{\mathrm{t}} = \{\Omega_{11}, \Omega_{22}, \Omega_{33}, 2\Omega_{23}, 2\Omega_{13}, 2\Omega_{12}, \chi_{11}, \chi_{22}, 2\chi_{12}\};$$

— for beams,

$$\{\varepsilon\}^{\mathrm{t}} = \{\Omega_{xx}, \Omega_{zz}, 2\Omega_{xz}, \chi_{xx}\};$$

$\{\sigma\}^{\mathrm{t}}$ is the vector of forces, moments, and shear forces:

— for shells and plates,

$$\{\sigma\}^{\mathrm{t}} = \{N_{11}, N_{22}, N_{33}, 2Q_{23}, 2Q_{13}, N_{12}, M_{11}, M_{22}, M_{12}\};$$

— for beams,

$$\{\sigma\}^{\mathrm{t}} = \{N_{11}, N_{33}, 2Q_{13}, M_{11}\}.$$

For the vectors of strains, forces, moments, and shear forces in the case of a gently sloping multilayer shell and multilayer plates, the 9×9 matrix of effective rigidities $[D]$ has the following form:

$$[D] = \begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} & 0 & 0 & \Theta_{16} & \Sigma_{11} & \Sigma_{12} & \Sigma_{16}\\
\Theta_{21} & \Theta_{22} & \Theta_{23} & 0 & 0 & \Theta_{26} & \Sigma_{21} & \Sigma_{22} & \Sigma_{26}\\
\Theta_{31} & \Theta_{32} & \Theta_{33} & 0 & 0 & 0 & \Sigma_{31} & \Sigma_{32} & 0\\
0 & 0 & 0 & \Theta_{44} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \Theta_{55} & 0 & 0 & 0 & 0\\
\Theta_{61} & \Theta_{62} & 0 & 0 & 0 & \Theta_{66} & 0 & 0 & \Sigma_{66}\\
\Sigma_{11} & \Sigma_{21} & \Sigma_{31} & 0 & 0 & 0 & \Xi_{11} & \Xi_{12} & \Xi_{16}\\
\Sigma_{12} & \Sigma_{22} & \Sigma_{32} & 0 & 0 & 0 & \Xi_{21} & \Xi_{22} & \Xi_{26}\\
\Sigma_{16} & \Sigma_{26} & 0 & 0 & 0 & \Sigma_{66} & \Xi_{61} & \Xi_{62} & \Xi_{66}
\end{bmatrix};$$

in the case of multilayer beams, the 4×4 matrix of effective rigidities $[D]$ has the form

$$[D] = \begin{bmatrix} \Theta_{11} & \Theta_{13} & 0 & \Sigma_{11} \\ \Theta_{31} & \Theta_{33} & 0 & \Sigma_{31} \\ 0 & 0 & \Theta_{55} & 0 \\ \Sigma_{11} & \Sigma_{31} & 0 & \Xi_{11} \end{bmatrix}.$$

Here the components of the matrices of effective rigidities for shells, plates, and beams are determined from Eqs. (5). Note that the matrices of effective rigidities are written for the case with the mid-surface of the layer being the plane of elastic symmetry in each layer of the CM packet.

2. Characteristics of CM Rigidity and Viscosity. The elastic properties of the entire CM packet are characterized by the layer rigidity matrix

$$[A^k] = \begin{bmatrix} A^k_{11} & A^k_{12} & A^k_{16} \\ A^k_{21} & A^k_{22} & A^k_{26} \\ A^k_{61} & A^k_{62} & A^k_{66} \end{bmatrix}. \qquad (6)$$

In turn, the components of the rigidity matrix are expressed via the components of a unidirectional monolayer (representative element) considered in the coordinate system $(1, 2)$ whose axes coincide with the axes of symmetry of the material (see Fig. 1):

$$[A^k] = [T_A]^{\rm t}[A_{1,2}][T_A].$$

Here $[A_{1,2}]$ is the matrix of rigidity of a unidirectional layer; in the case of a plane stress state, this matrix has the form [5]

$$[A_{1,2}] = \frac{1}{1 - \nu_{12}\nu_{21}} \begin{bmatrix} E_{11} & \nu_{21}E_{11} & 0 \\ \nu_{12}E_{22} & E_{22} & 0 \\ 0 & 0 & (1 - \nu_{12}\nu_{21})G_{12} \end{bmatrix};$$

$[T_A]$ is the rotation matrix for the matrix of rigidity of the unidirectional CM:

$$[T_A] = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix}; \qquad (7)$$

$s = \sin\varphi$, $c = \cos\varphi$, and φ is the angle of packing of the fibers.

Carbon plastics and fiberglass plastics are layered fiber-reinforced CMs, which possess viscoelastic properties to a smaller or greater extent owing to the presence of a polymer binder. As Rheological relations [13], we use the linear dependence of stresses on strains and strain rates (Voigt model):

$$\sigma^{ij} = A^{ijkl}e_{kl} + W^{ijkl}\,\frac{de_{kl}}{dt} \qquad (i, j, k, l = 1, 2, 3).$$

Here σ^{ij} are the components of the stress tensor of the layer, e_{kl} are the components of the strain tensor, A^{ijkl} are the components of the rigidity tensor, and W^{ijkl} are the components of the viscosity tensor of the layer.

The components of the rigidity matrix of the layer are determined by Eq. (6). Let us consider one possible method of determining the components of the viscosity matrix. The matrix of decay decrements is normally used as the viscosity matrix [14]. Based on the analysis of specific features of formation of dissipative characteristics of layered fiber-reinforced CMs under dynamic loading, Zinov'ev and Ermakov [15] derived an elastic-dissipative matrix, which is used as the viscosity matrix in the present work. According to [15], for a unidirectional CM considered in the coordinate system $(1, 2)$ whose axes coincide with the axes of symmetry of the material (see Fig. 1), the viscosity matrix of a unidirectional monolayer in the case of a plane stress state is written as

$$[W_{1,2}] = \begin{bmatrix} \psi_{11}/E_{11} & -\nu_{21}\psi_{11}/E_{22} & 0 \\ -\nu_{12}\psi_{11}/E_{11} & \psi_{22}/E_{22} & 0 \\ 0 & 0 & \psi_{12}/G_{12} \end{bmatrix} \qquad (8)$$

(ψ_{11}, ψ_{22}, and ψ_{12} are the dissipation coefficients determined in [15] in the case of free oscillations of cantilever samples made of carbon plastic).

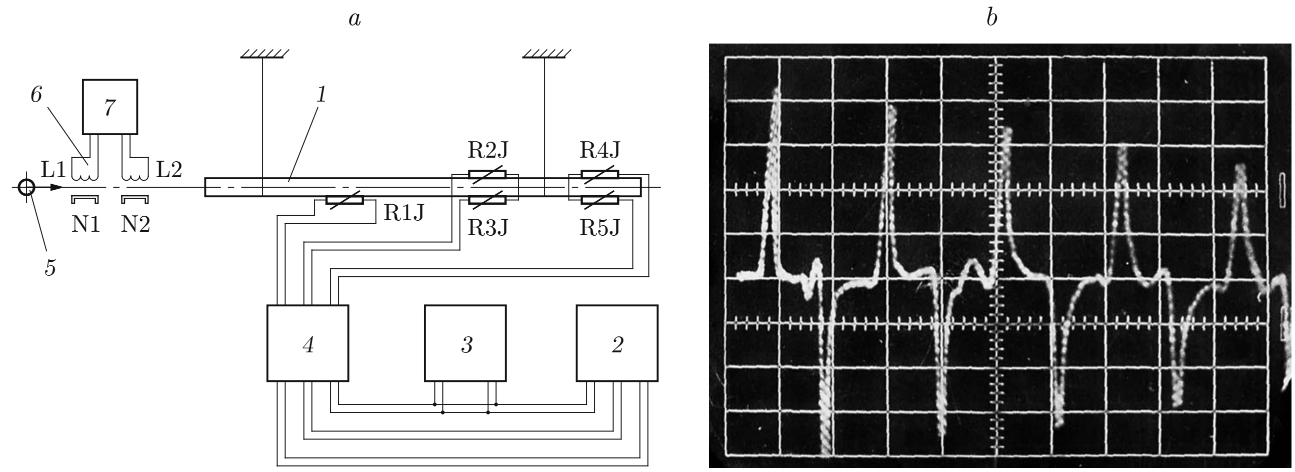

Fig. 2. Layout of the experimental facility (a) and typical oscillogram (b) used to determine the dynamic characteristics of rigidity by the method of an "unsteady rod": 1) rod to be tested; 2) S8-13 electron oscillograph with memory; 3) ChZ-34 electron frequency meter for determining the wave travel time; 4) Ya40-1103 (1U14) high-sensitivity amplifier; 5) accelerated solid (impactor); 6) magnetoinductive gauges for recording the impactor velocity; 7) ChZ-34 electron frequency meter for determining the impactor velocity.

If the coordinate system is rotated by an angle φ (see Fig. 1), the viscosity matrix of a unidirectional monolayer in the coordinate system (x, y) acquires the form

$$[W^k] = [T_B]^{\mathrm{t}}[W_{1,2}][T_B]. \tag{9}$$

Here $[T_B]$ is the rotation matrix for the viscosity matrix of the unidirectional CM. The rotation matrix for the viscosity matrix (9) is denoted by T_B, because it differs from a similar rotation matrix (7) for the rigidity matrix of the unidirectional CM:

$$[T_B] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}.$$

Thus, the viscous properties of the entire packet of the composite is characterized by the viscosity matrix of the layer

$$[W^k] = \begin{bmatrix} W_{11}^k & W_{12}^k & W_{16}^k \\ W_{21}^k & W_{22}^k & W_{26}^k \\ W_{61}^k & W_{62}^k & W_{66}^k \end{bmatrix}.$$

The determining components of the viscosity matrix are E_{11}, E_{22}, and G_{12}, which are commonly accepted engineering elastic constants, and the coefficients ψ_{11}, ψ_{22}, and ψ_{12}, which are engineering viscous constants obtained from experiments with simple forms of the stress state.

3. Dynamics Elastic and Viscous Constants of the Composite Material. For determining the dynamic elasticity modulus E_{11}^{dyn}, a method was developed in [10, 11], which is based on registration of the velocity of propagation of incident and reflected strain waves after a longitudinal impact on the end face of a freely suspended rod made of a composite material. This method was called the method of an "unsteady rod." The layout of the experimental facility and the typical oscillogram are shown in Fig. 2.

KF-4P1-5-100-B-12 strain gauges are glued onto the freely suspended rod in three cross sections. These gauges are connected to a high-sensitivity amplifier through a bridging measurement circuit. An R1J strain-gauge transducer is applied to synchronize the moment of actuation of the oscillograph curve with the moment of the action of the longitudinal wave on the R2J (or R3J) and R4J (or R5J) strain-gauge transducers. Symmetric alignment of the strain-gauge transducers on the rod is necessary to eliminate the flexural component of the longitudinal wave. The impact was performed on the end face of a $330 \times 10 \times 10$ mm rod made of fiberglass plastic, with packing angles of 0° and 90°. The impactor mass was $m = 3 \cdot 10^{-3}$ kg, and the impact velocity was $V = 40$ m/sec. The

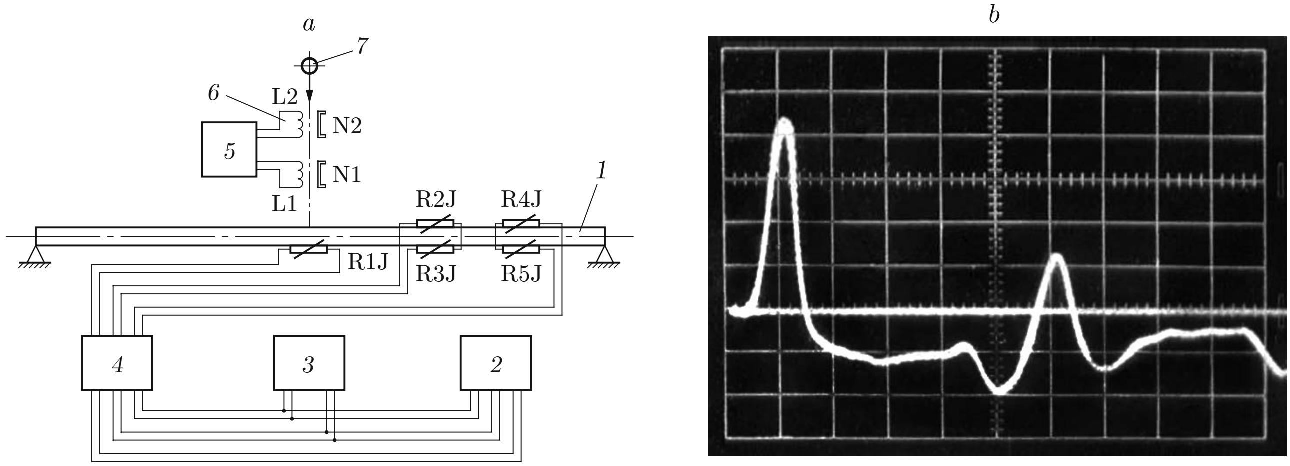

Fig. 3. Layout of the experimental facility (a) and typical oscillogram (b) used to determine the dynamic characteristics of rigidity by the method of an "unsteady beam": 1) beam to be tested; 2) S8-13 electron oscillograph with memory; 3) ChZ-34 electron frequency meter for determining the wave travel time; 4) Ya40-1103 (1U14) high-sensitivity amplifier; 5) ChZ-34 electron frequency meter for determining the impactor velocity; 6) magnetoinductive gauges for recording the impactor velocity; 7) accelerated solid (impactor).

impactor velocity was registered by magnetoinductive gauges, as the impactor covered the basic distance between the inductance coils L1 and L2 with the magnets N1 and N2.

The oscillogram plotted in Fig. 2b shows the propagation of the pulse of the longitudinal wave along the rod with its multiple reflection from the end faces with subsequent decay. The experimentally determined velocity of propagation of the longitudinal strain wave $C_E^{\rm exp}$ and the time $\Delta\tau$ needed for the longitudinal wave to cover the basic distance L determined from the oscillogram are related as

$$C_E^{\rm exp} = L/\Delta\tau.$$

The dynamic longitudinal elasticity modulus is determined by the formula

$$E_{11}^{\rm dyn} = \rho (C_E^{\rm exp})^2,$$

where ρ is the density of the composite material of the rod being tested.

Based on the analysis of transitional processes in CM beams under a transverse impact [11, 16], it was shown that oscillograms obtained under certain conditions imposed on the values of the impact pulse and characteristic size of the beams provide the incident and reflected flexural strain waves propagating with a velocity of the shear wave. This wave velocity, in turn, is functionally related to the dynamic shear modulus $G_{13}^{\rm dyn}$. The proposed method of determining the dynamic shear modulus was called the method of an "unsteady beam."

Figure 3 shows the layout of the experimental facility and the typical oscillogram of propagation of the flexural pulse. KF-4P1-5-100-B-12 strain gauges are glued onto the clamped beam in three cross sections. These gauges are connected to a high-sensitivity amplifier through a bridging measurement circuit. An R1J strain-gauge transducer is applied to synchronize the moment of actuation of the oscillograph curve with the moment of the action of the flexural wave on the R2J (or R3J) and R4J (or R5J) strain-gauge transducers. The transverse impact was performed on the end face of the same sample that was previously loaded by a longitudinal impact by the "unsteady rod" arrangement with the same impact parameters.

The oscillogram plotted in Fig. 3b shows the propagation of the incident and reflected flexural pulses along the beam with subsequent decay. The experimentally determined velocity of propagation of the flexural wave and the time $\Delta\tau$ needed for the flexural wave to cover the basic distance L are related as

$$C_G^{\rm exp} = L/\Delta\tau.$$

The dynamic shear modulus is determined by the formula

$$G_{13}^{\rm dyn} = \rho (C_G^{\rm exp})^2/k.$$

TABLE 1

Static and Dynamic Elastic Characteristics of Composite Materials Being Tested

CM	φ, deg	$E_{11}^{\text{dyn}} \cdot 10^{-8}$, N/m^2	$G_{13}^{\text{dyn}} \cdot 10^{-8}$, N/m^2	$E_{11}^{\text{dyn}}/E_{11}^{\text{st}}$	$E_{11}^{\text{dyn}}/G_{13}^{\text{dyn}}$
Fiberglass plastic	0, 90	283	47	1.11	6.0
	$\pm$45	174	38	1.08	4.6
	0, $\pm$45, 90	223	43	1.09	5.2
Carbon plastic	0, 90	1480	88	1.08	16.9
	$\pm$45	1154	81	1.04	14.3
	0, $\pm$45, 90	1325	85	1.06	15.5

TABLE 2

Dynamic Viscous Characteristics of Composite Materials Being Tested

CM	φ, deg	W_{11}^{dyn}, m^2/N	W_{22}^{dyn}, m^2/N	W_{12}^{dyn}, m^2/N
Fiberglass plastic	0, 90	0.260	0.257	0.138
	$\pm$45	0.290	0.294	0.117
	0, $\pm$45, 90	0.271	0.273	0.130
Carbon plastic	0, 90	0.072	0.073	0.039
	$\pm$45	0.093	0.094	0.027
	0, $\pm$45, 90	0.081	0.082	0.031

Here ρ is the density of the material of the beam made of an CM and k is a coefficient depending on the cross-sectional shape (for a beam with a rectangular cross section, $k = 2/3$).

Figures 2b and 3b show the oscillograms of propagation of the longitudinal and flexural pulses for fiberglass plastic rods and beams, respectively. Similar oscillograms were obtained for carbon plastic rods and beams whose characteristics are presented below. The fiberglass plastic monolayer is characterized by $\rho = 1800$ kg/m^3, $h = 0.3 \cdot 10^{-3}$ m, $E_{11} = 270 \cdot 10^8$ N/m^2, $E_{22} = 70 \cdot 10^8$ N/m^2, $G_{12} = 46 \cdot 10^8$ N/m^2, and $\nu_{12} = 0.26$. The carbon plastic monolayer is characterized by $\rho = 1450$ kg/m^3, $h = 0.125 \cdot 10^{-3}$ m, $E_{11} = 1400 \cdot 10^8$ N/m^2, $E_{22} = 85 \cdot 10^8$ N/m^2, $G_{12} = 61 \cdot 10^8$ N/m^2, and $\nu_{12} = 0.28$. The volume fraction of the polymer binder in the CM packet is 35%.

The effective elasticity and shear moduli for the CM packet E_{11}^{st}, E_{22}^{st}, and G_{13}^{st}, calculated by the dependences derived in [5] will be called the static elasticity and shear moduli. The dynamic elasticity and shear moduli E_{11}^{dyn}, E_{22}^{dyn}, and G_{13}^{dyn} are determined on the basis of the experimental "unsteady" methods: the method of an "unsteady rod" and the method of an "unsteady beam" from the velocities of propagation of the longitudinal and flexural strain waves.

The results of determining the static and dynamic elasticity moduli of the CMs being tested are summarized in Table 1, which shows that the dynamic elasticity moduli are higher than the corresponding static moduli by 8–11% for fiberglass plastic and by 4–8% for carbon plastic. The greatest difference is observed for fiberglass plastic reinforced at angles 0° and 90°; the smallest difference is observed for carbon plastic reinforced at angles $\pm 45°$. The dynamic moduli retain a high degree of anisotropy: $E/G \approx 6$ for fiberglass plastic and $E/G \approx 15$ for carbon plastic.

Let us introduce dynamic moduli of viscosity and moduli of shear viscosity W_{11}^{dyn}, W_{22}^{dyn}, and W_{13}^{dyn}, which depend on the dynamic elasticity and shear moduli and are determined by Eqs. (8) and (9) on the basis of the experimentally obtained velocities of propagation of the longitudinal and flexural strain waves and their decay decrements.

For determining the moduli of viscosity of longitudinal elasticity W_{11}^{dyn} and W_{22}^{dyn}, we tested the same $330 \times 10 \times 10$ mm rods and beams with a rectangular cross section. In contrast to the experiments described in [15], we used the oscillograms of post-impact oscillations obtained by the method of an "unsteady rod" (see Fig. 2b). The time sweep was chosen so that the oscillograph screen showed at least 10 cycles of travelling of the longitudinal wave over the rod. The results of oscillogram processing for the CMs being tested are shown in Table 2. The longitudinal viscosity moduli W_{11}^{dyn} and W_{22}^{dyn} were obtained by the method of an "unsteady rod." It is difficult to use the method of an "unsteady beam" for determining the shear viscosity modulus W_{13}^{dyn} because of a strong dispersion of the flexural pulse propagating over the beam. Therefore, the values of the shear viscosity modulus given in Table 2 were obtained by the method described in [15], i.e., based on the logarithmic decay decrements in the directions of the fiber packing angles 0°, 90°, and 45°. It follows from Table 2 that the degree of damping of fiberglass plastic is higher than the degree of damping of carbon plastic with an identical volume fraction of the polymer binder. For materials reinforced at angles 0° and 90°, the damping capability of fiberglass plastic is four times that of carbon plastic. The highest damping capability in both fiberglass plastic and carbon plastic is observed in the case of their reinforcement at angles ±45°, as compared with their damping capabilities in samples with other types of reinforcement.

Conclusions. In a theoretical analysis of impact loading and unsteady deformation of composite beams, plates, and shells with rather simple rheological relations, such as the Voigt model, used as physical relations, the determining components of the viscosity matrix can be the dynamic viscosity moduli, as well as the dynamic elasticity and shear moduli, which are obtained in experiments with fairly simple types of the stress state.

The dynamic elasticity and shear moduli $E_{11}^{\rm dyn}$, $E_{22}^{\rm dyn}$, and $G_{13}^{\rm dyn}$ are determined from the velocities of propagation of the longitudinal and flexural strain waves with the use of experimental "unsteady methods."

The dynamic viscosity and shear viscosity moduli $W_{11}^{\rm dyn}$, $W_{22}^{\rm dyn}$, and $W_{13}^{\rm dyn}$ depend on the dynamic elasticity and shear moduli and are determined from the experimentally obtained velocities of propagation of the longitudinal and flexural strain waves and their decay decrements.

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